Mike Weiss, with Brian Dodge, Kate Harden, Amy Hempstead, Jeff Lloyd, and Brian Pott

Using a Model Rocket-Engine Test Stand in a Calculus Course

Advanced Placement calculus students from Yarmouth High School, located in Yarmouth, Maine, built a model rocket-engine test stand in May 2000. This project was used to reinforce calculus concepts. In addition, it furnished data for a future freshman-science rocketry project. Students used the concepts of the average value of a function, along with Riemann-sum approximation techniques and spreadsheet skills to determine the average thrust of a model rocket engine.

BACKGROUND

The rocketry unit in freshman science focuses on laws of motion and on predicting the maximum height of the students' rocket. The unit begins with discussions and investigations regarding Newton's laws and then addresses predicting rocket altitudes by using preflight data. An important piece of preflight data is the value of the average thrust of the rocket's engines.

The objectives of the project for the AP calculus students were to—

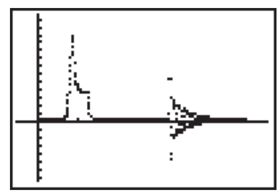
- emulate scientists in solving an engineering problem;
- apply calculus principles to the applications;
- gather experimental data on model rocket engines;
- develop a thrust curve that a calculus class can analyze; and
- determine the average thrust of a model rocket engine.

The thrust curve is a plot of force versus time

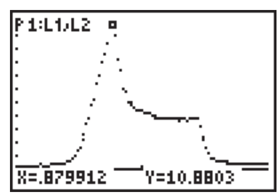
ROCKET ENGINE THRUST

According to *Handbook of Model Rocketry* (National Aeronautics and Space Administration 1996), the thrust of a rocket engine is the amount of force, or push, produced when it is operating. Newton's third law of motion indicates that the jet of gas rushing out of the nozzle produces a force because of conservation of momentum. Stated simply, for every acting force, there is an equal and opposite directed reacting force. The thrust of a model rocket engine is rarely constant. It changes with time. Several aspects of the thrust curve correspond to different stages of the engine's performance. *Maximum thrust* is the highest amount of force produced by the engine during operation, regardless of when it occurs during the period of operation. This maximum thrust produces a "peak" on the thrust curve, as shown in **figure 1.** The thrust curve for a model rocket engine is a plot of force versus time, as shown in **figure 2.**

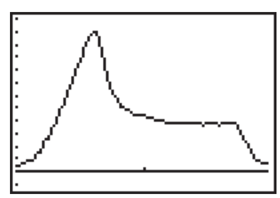
Average thrust is a derived, or calculated, number. It is determined by dividing the total impulse, that is, the product of force and time, by the duration. It indicates what the thrust would be if it were



C 6-5 engine test firing showing thrust and ejection charge



C 6-5 engine thrust curve (peak thrust highlighted)



C 6-5 engine thrust curve

Fig. 1
TI-83 screen captures

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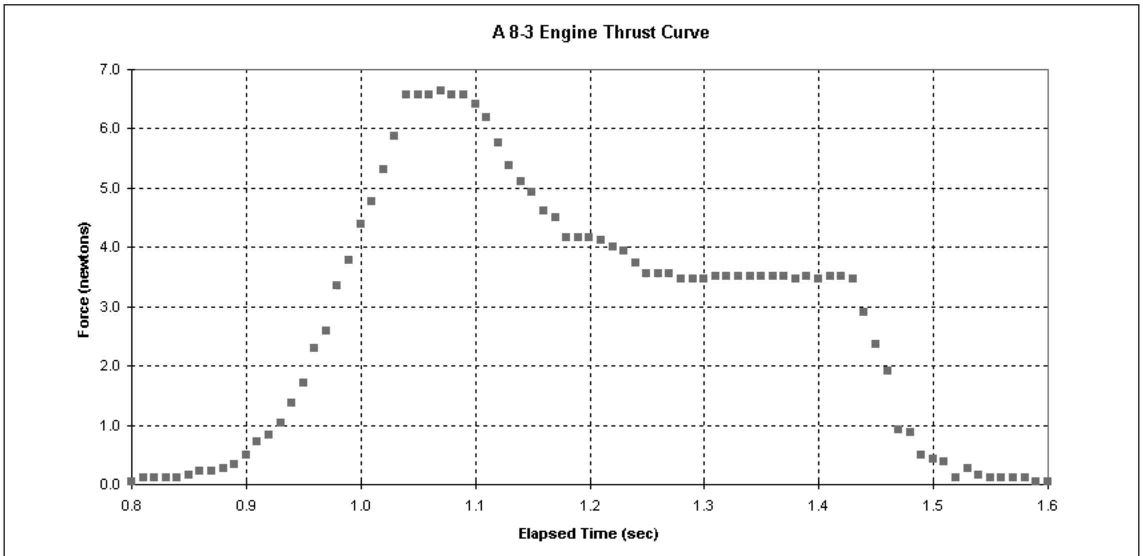


Fig. 2

Average thrust is an essential piece of information for altitude prediction

constant from ignition to burnout. Mathematically, the average value of a continuous function is

$$\frac{\int_a^b f(t)dt}{b-a}.$$

In this example, it is the area under the thrust curve divided by the length of time that the propellant burns. From a graphical standpoint, it corresponds to a constant function on the same interval that has the same area as the thrust curve. See **figure 3.** Average thrust is an essential piece of information in predicting altitude. It also indicates how much the model rocket will accelerate and roughly how fast it will be going when it leaves the launch rod.

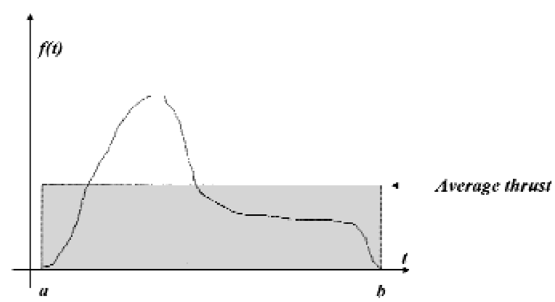


Fig. 3
For an equal time interval, a to b, the area under the thrust curve is the same as the area of a rectangle with a height equal to the average thrust.

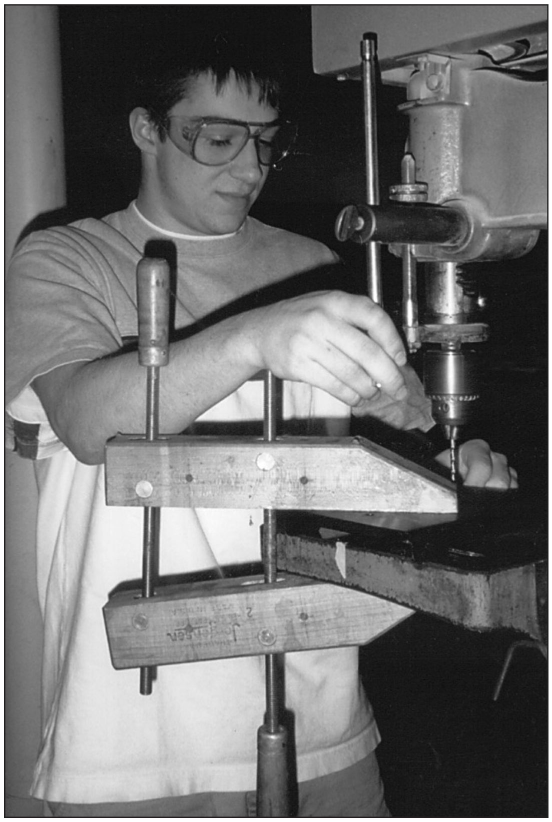
DESIGNING AND BUILDING THE TEST STAND

To support the freshman-science rocketry unit, the AP calculus class was asked to design an apparatus that could collect the necessary data to construct a thrust curve generated by a commercially produced model-rocket engine. Available equipment included a Texas Instruments Calculator-Based Laboratory (CBL) and a Vernier Student Force Sensor (20 newtons maximum force). The students began to brainstorm ideas so that they could mount these instruments into a test stand. The test stand had to be inexpensive, easy to operate, and portable; and the data had to be accurate.

The students became familiar with the capabilities of the CBL, the Force Sensor, and **program 1,** which was written for the TI-83 graphing calculator. Of major concern was the sensitivity of the force sensor, as well as whether enough data points could be collected in the very brief engine burn time. An "A" size engine has a propellant burn time of less than one second. In solving this problem, the class revisited a popular physics-course activity that used simple harmonic motion. It involved a known mass and spring, along with other equipment. The results of the activity showed that students could collect enough data points during the burn time. **Program 1** initially came with the force sensor and was then modified for use in collecting thrust data.

The students next needed to design a stand that could safely hold the equipment for collecting the data. Working with the industrial technology teacher, they discussed various design ideas and noted the strengths and weaknesses of each. Some of the students had also participated in the National Engineering Design Challenge and had already operated such equipment as drills and welding equipment, which they needed to use in the construction phase of the project.

To hold the engine in place during the test launch, the students used hose clamps and screws to attach a piece of steel electrical conduit (pipe) to a ring



A student drills holes in the metal base.

The students discussed various design ideas

stand. They welded the ring stand to a steel plate for additional stability and treated the apparatus with heat-resistant paint. The force sensor was attached to the rod on the ring stand. The students solved the problem of transferring the force of the engine to the force sensor without damaging the sensor by epoxying the head of a nail to a penny to create a plunger. The engine pressed on the penny, which transferred the force via the nail to the sensor. The penny fit nicely into the pipe, and the diameter of the penny closely approximated the diameter of the engine. Students fabricated a support for the plunger from a piece of sheet metal, which also acted as a blast shield and thereby protected the force sensor from the engine's ejection charge. The entire steel base plate was screwed to two pieces of wood that elevated the apparatus off the ground.

COLLECTING AND TRANSFERRING THE DATA

The students connected the CBL to the force sensor with a fifteen-foot cable so that the operator could stand a safe distance away from the test stand and collect data. A TI-83 graphing calculator connected to the CBL could collect data at a rate of 100 data points per second. The engine was electrically fired using an igniter and launch system familiar to model-rocket enthusiasts. Students stored the data points as a list in the calculator. The students easi-

PROGRAM 1
Rocket Engine Data Collection Program (TI-83)

```
ClrHome
ClrDraw
FnOff
512→dim(L1)                        // Set L1 to 512 items (would
                                   otherwise be 199)
512→dim(L2)                        // Set L2 to 512 items (would
                                   otherwise be 199)
{1,0}→L6
Send(L6)
{1,1,1,0,0,1}→L6
Send(L6)
Disp "PRESS [ENTER]"               // Press when ready for the
                                   CBL to start
Pause
{4,1,1,1,-9.8,9.8}→L6              // (see note 1)
Send(L6)
{3,.010,512,0,0,0,0,0,1}→L6        // (see note 2)
Send(L6)
Get(L2)
Get(L1)

Plot1(Scatter,L1,L2,•)             // Plots the results (time is on
                                   the x axis)

Stop

// Note 1:
// The last two numbers are the calibration numbers for the
sensor we used.
// To reverse which way is positive, just switch the positive
number with the negative number.

// Note 2:
// The second number is the time interval in seconds. In this
program, the sensor takes a reading every .01 seconds. The
third number is the number of readings that the CBL will
take; here, 512 is the maximum that the CBL will take. Make
sure that it corresponds with the size of the list.
// This program is a modification of the sample program that
accompanies the Vernier Student Force Sensor.
```

ly downloaded the list to a computer using a Graph Link and then transferred it to Excel. To transfer the list to Excel, the students first opened the list in the Graph Link software; then they selected all the elements (Edit, Select All). They then copied and pasted data points into the Excel spreadsheet.

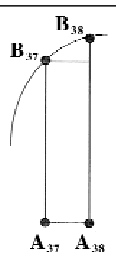
The thrust curve of a single-stage model-rocket engine, shown in the graphing-calculator screen captures in **figure 1,** illustrates the large initial push, which then drops to a level, sustained amount until the engine burns out. The figure shows the region of the delay charge burns (no thrust) and the point at which the ejection charge fires. The ejection charge shows up as a positive and negative deflection of the sensing bar on the force probe.

Once the data points were transferred to Excel, students could make a thrust curve showing the force versus time. The impulse, that is, the area under the curve, can be determined by a variety of integration techniques. **Figure 2** was made by using the graphing features of Excel with the data transferred from the graphing calculator. It represents the region where thrust was taking place (an elapsed time of 0.8 sec. to 1.8 sec.). The portion of the delay and ejection charges have not been plotted. The students did not have an equation for the thrust curve, but they used data points of engine thrust collected at a rate of 100 per second. To solve for the area under the thrust curve, the students used the Riemann-sum technique of dividing the area into very thin rectangles, solving for the area of each rectangle, and then combining the areas to yield an approximation for the total area under the thrust curve. Students calculated the areas of the thin rectangles by using the force measurements taken (heights) and multiplying each one by the time between measurements (0.01 sec.). The equation for the Riemann sum is

$$R_n = \sum_{k=1}^{n} f(t_k)\Delta t_k,$$

where $f(t_k)$ is the force reading on the left-hand side of the rectangle and Δt is the time between measurements. For example, a rocket engine that burned for 1.5 seconds would yield 150 rectangular areas to compute and combine. A spreadsheet is well-suited for this repetitive task. Average thrust is then determined by dividing the impulse by the duration of engine burn time. Students easily set up the computation using a Microsoft Excel spreadsheet. **Figure 4** shows a graphical example of a rectangle under the thrust curve and associated calculation in an Excel spreadsheet. Using a reading taken on the right-hand side of the rectangle gave essentially the same results because the rectangles were sufficiently narrow.

The average of several firings of "A" engines yielded an average thrust of 3.27 newtons. These results compared very favorably with the figure of 3.18 newtons published by the National Association of Rocketry (NAR) as the average for similar engines. The NAR standards and testing results are available for viewing at www.thrustcurve.org, along with other data regarding model rocket engines. The science students can then use this information to help calculate the predicted altitude of a model rocket.



	A	B	C
	time (sec)	force (N)	area (N-sec)
37	1.06	6.583	0.06583
38	1.07	6.638	0.06638

where cell C37 = (A38 – A37)*B37, that is, the area of the thin rectangle calculated in C37 is the width (A38 – A37) multiplied by the height of the left side (B37)

Fig. 4
A graphical example of a thin rectangle under the thrust curve and associated calculation in an Excel spreadsheet to generate the Riemann sum

In summary, calculus students used their mathematics skills in a physics setting to solve a real-world engineering problem. The concept of the average value of a continuous function is covered in high school calculus but is often too abstract for first-year calculus students. Students needed to design a piece of equipment that allowed them to collect data points and use methods similar to those used by scientists and engineers. This project required an understanding of the value of Riemann sums as an integration approximation technique and led toward the use of finding the average value of a function. Although available software can determine the area under a curve given a function or can determine associated sets of data points, this project challenged students to write or modify a program for the CBL and use a spreadsheet to solve this problem. This project gives a practical application of these concepts and shows how students can be actively involved in developing the concepts.

EXTENSION FOR A FRESHMAN SCIENCE COURSE

Various companies have developed probes and sensors that can be easily adapted for use in a high school science class. Readily available programs for computers and graphing calculators do not require that teachers or students be well-versed in programming. In fact, an excellent source for free programs can be found at Vernier's Web site, www.vernier.com. This site allows users to download and quickly use powerful applications on a variety of TI graphing calculators. The programs will do the mathematics for which spreadsheets were used in this project. In particular, the Physics group of programs for the TI-83 and the Physics App for the TI-83 Plus include an Analyze option that approximates the area under the curve of the graph for the collected data. These programs could be an ideal way to introduce exciting calculus concepts to freshmen science students.

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BIBLIOGRAPHY

Foerster, Paul A. *Calculus: Concepts and Applications*. Berkeley, Calif.: Key Curriculum Press, 1998.

National Aeronautics and Space Administration. *Rockets: A Teacher's Guide with Activities in Science, Mathematics, and Technology.* Washington, D.C.: Office of Human Resources and Education, 1996.

Stine, G. Harry. *Handbook of Model Rocketry*. New York: John Wiley & Sons, 1994.